K03 ACCELERATION DUE TO GRAVITY AND PROJECTILE MOTION

SPH4U

CH 1 (THE BIG PICTURE)

- the linear motion of objects in horizontal, vertical, and inclined planes
- the motion of a projectile in terms of components of its motion
- objects moving in two dimensions
- predict the motion of an object
- technological devices based on the concepts and principles of projectile motion

EQUATIONS

- Acceleration Due to Gravity

$$\vec{g} = 9.8\ \mathrm{m/s^2}[\mathrm{down}]$$

- Vector Components

$$V_x = V\cos\theta$$
$$V_y = V\sin\theta$$

ACCELERATION DUE TO GRAVITY

- **Acceleration due to Gravity ($\vec{g}$):** the acceleration of an object falling vertically toward Earth's surface
- **Free Fall:** when air resistance is negligible
- All objects fall with the same acceleration when neglecting air resistance
- Acceleration due to gravity on Earth:

$$g = 9.8\ \mathrm{m/s^2}$$

TERMINAL VELOCITY

- Terminal Velocity (or Speed): the maximum speed of a falling object at which point the speed remains constant and there is no further acceleration
  - occurs when air resistance balances the acceleration due to gravity
- Sky divers use parachutes to decrease their terminal velocity

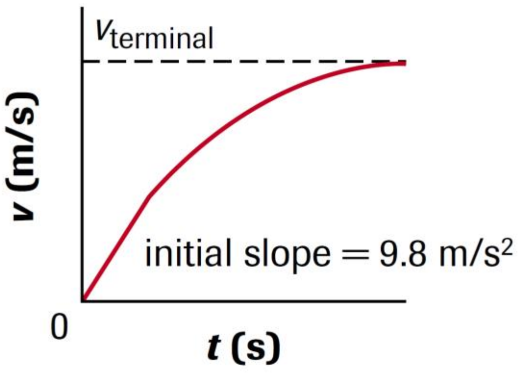
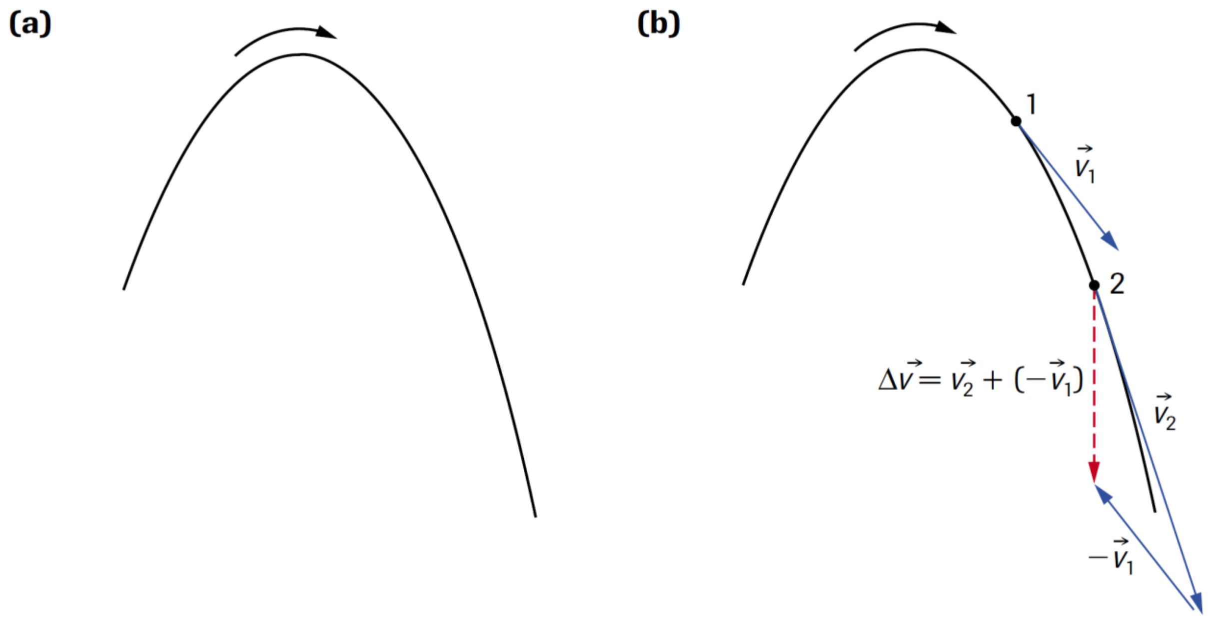


Figure 6
The general shape of a speed-time graph for a falling object that reaches terminal speed

PROJECTILE MOTION

- **Projectile:** an object that moves through the air, along a trajectory, without a propulsion system
- **Projectile Motion:** the motion of a projectile



PROJECTILE MOTION

- Constant horizontal velocity combined with a constant vertical acceleration due to gravity
  - Do not have vector symbol because they are components
- Since the vectors are perpendicular, we can analyse them separately using the kinematic equations

PROJECTILE MOTION EQUATIONS

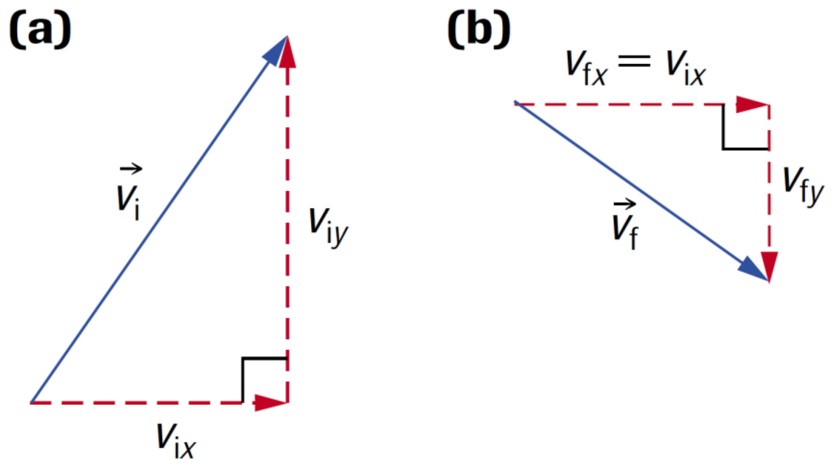
- Horizontal: $v_x = v_{ix} = v_{fx} = \frac{\Delta d_x}{\Delta t}$
- Vertical: $v_{fy} = v_{iy} + a_y \Delta t$

$$\Delta d_y = v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$\Delta d_y = v_{fy}\Delta t - \tfrac{1}{2}a_y(\Delta t)^2$$

$$\Delta d_y = \tfrac{1}{2}\left(v_{iy} + v_{fy}\right)\Delta t$$

$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta d_y$$



PROBLEM 1

Dan pushes Mike off the roof of the school (8.0 m high) with no initial velocity.

a) How long does it take Mike to land?

PROBLEM 1 – SOLUTIONS

a) Let up be positive.

	x	y
d_i	0	8 m
d_f	0	0
v_i	0	0
v_f	0	?
a	0	-9.8 m/s^2
Δt	?	?

$$\Delta d_y = -8.0 \text{ m}$$

$$\Delta d_y = v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$

$$= \sqrt{\frac{2(-8.0)}{-9.8}}$$

$$= 1.3 \text{ s}$$

∴ it takes Mike 1.3 s to land.

PROBLEM 2

Mike, unimpressed with Dan, pushes Dan off the roof, more forcefully, with a horizontal velocity of 5.0 m/s.

a) How long until Dan lands?

b) How far away from the school does he land?

PROBLEM 2 – SOLUTIONS

a) Let up be positive.

	x	y
d_i	0	8 m
d_f	?	0
v_i	5.0 m/s	0
v_f	5.0 m/s	?
a	0	-9.8 m/s^2
Δt	?	?

$$\Delta d_y = -8.0 \text{ m}$$

$$\Delta d_y = v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$

$$= \sqrt{\frac{2(-8.0)}{-9.8}}$$

$$= 1.3 \text{ s}$$

∴ it takes Dan 1.3 s to land.

PROBLEM 2 – SOLUTIONS CONT.

b) Let forward be positive.

	x	y
d_i	0	8 m
d_f	?	0
v_i	5.0 m/s	0
v_f	5.0 m/s	?
a	0	-9.8 m/s^2
Δt	1.3 s	1.3 s

$$v_x = \frac{\Delta d_x}{\Delta t}$$

$$\begin{aligned}\Delta d_x &= v_x \Delta t \\ &= (5.0\ \mathrm{m/s})(1.3\ \mathrm{s}) \\ &= 6.5\ \mathrm{m}\end{aligned}$$

∴ Dan lands 6.5 m away from the school.

PROBLEM 3

Dan, not liking his friend Mike at all now, throws Mike from the roof with a velocity of 7.0 m/s at an elevation of 25° above the horizon.

a) How long until Mike lands?

b) How high above the *ground* did he reach?

c) How far from the school does he land?

PROBLEM 3 – SOLUTIONS CONT.

a) Let up be positive.

	x	y
d_i	0	8m
d_f	?	0
v_i	7.0 cos 25° m/s	7.0 sin 25° m/s
v_f	7.0 cos 25° m/s	?
a	0	-9.8 m/s²
Δt	?	?

$\Delta d_y = -8.0\text{ m}$

∴ it is 1.6 s until Mike lands.

$$\Delta d_y = v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$\Delta t = \frac{-(v_{iy}) \pm \sqrt{{v_{iy}}^2 - 4\left(\frac{1}{2}a_y\right)(-\Delta d_y)}}{2\left(\frac{1}{2}a_y\right)}$$

$$= \frac{-(7.0\sin 25°) \pm \sqrt{(7.0\sin 25°)^2 - 4\left(\frac{1}{2}(-9.8)\right)(-(-8.0))}}{2\left(\frac{1}{2}(-9.8)\right)}$$

$$= -0.97\text{ s}, 1.6\text{ s}$$

$$t \geq 0 \Rightarrow \Delta t = 1.6\text{ s}$$

PROBLEM 3 – SOLUTIONS

b) Let $d_{max,y}$ be the max height above the school roof, Δt_{max} be the time this height is reached.

$\vec{v}_i = 7.0\ m/s$ [fwd 25° up]

	x	y
d_i	0	8 m
d_f	?	0
v_i	7.0 cos 25° m/s	7.0 sin 25° m/s
v_f	7.0 cos 25° m/s	?
a	0	-9.8 m/s²
Δt	1.6 s	1.6 s
d_{max}	-	?
v_{max}	7.0 cos 25° m/s	0
Δt_{max}	?	?

$$v_{max,y} = v_{iy} + a_y \Delta t_{max}$$

$$\Delta t_{max} = \frac{v_{max,y} - v_{iy}}{a_y} = \frac{0 - 7.0 \sin 25°}{-9.8} = 0.30\text{ s}$$

$$d_{max,y} = v_{iy}\Delta t_{max} + \tfrac{1}{2}a_y(\Delta t_{max})^2 = (7.0 \sin 25°)(0.30) + \tfrac{1}{2}(-9.8)(0.30)^2 = 0.45\text{ m}$$

$$\Delta d_{max,y} = \Delta d_y + d_{max,y} = 8.0\text{ m} + 0.45\text{ m} = 8.5\text{ m}$$

$\therefore$ Mike reaches a height of 8.5 m above the ground

PROBLEM 3 – SOLUTIONS

c)

	x	y
d_i	0	8 m
d_f	?	0
v_i	$7.0\cos 25°$ m/s	$7.0\sin 25°$ m/s
v_f	$7.0\cos 25°$ m/s	?
a	0	-9.8 m/s^2
Δt	1.6 s	1.6 s

$$\begin{aligned}\Delta d_x &= v_{ix}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \\ &= (7.0\cos 25°)(1.6) \\ &= 10\text{ m}\end{aligned}$$

∴ Mike falls exactly 10 m away from the school.

PROBLEM 4

Too tired to climb back onto the roof and calmly settle his differences with his former friend Dan, Mike kicks a soccer ball that hits Dan with a horizontal velocity of 12.0 m/s. If he kicked the ball at an angle of 30° above the horizon:

a) What was the initial velocity of the ball?

b) How far away was Mike when he kicked the ball?

Note: No Mikes or Dans were injured during the making of any of these questions and since this incident, the two have reconciled their differences and remain both good friends, and more importantly, good physics students.

PROBLEM 4 – SOLUTIONS

a)

	x	y
d_i	?	0
d_f	0	8 m
v_i	12.0 m/s	?
v_f	12.0 m/s	0
a	0	-9.8 m/s^2
Δt	?	?

$$v_{fy}^2 = v_{iy}^2 + 2a_y\Delta d_y$$

$$v_{iy} = \pm\sqrt{-2a_y\Delta d_y}$$

$$= \pm\sqrt{-2(-9.8)(8.0)}$$

$$= \pm 13\text{ m/s, up so} > 0$$

$$= 13\text{ m/s}$$

$$|\vec{v}_i| = \sqrt{(v_{ix})^2 + (v_{iy})^2}$$

$$= \sqrt{(12.0)^2 + (13)^2}$$

$$= 18\text{ m/s}$$

$$\Delta d_y = 8.0\text{ m}$$

$$\theta = 30°$$

$$\therefore \vec{v}_i = 18\text{ m/s [30° up from horizontal]}$$

PROBLEM 4 – SOLUTIONS

b)

	x	y
d_i	?	0
d_f	0	8 m
v_i	12.0 m/s	?
v_f	12.0 m/s	0
a	0	-9.8 m/s^2
Δt	?	?

$$\Delta d_y = v_{fy}\Delta t - \tfrac{1}{2}a_y(\Delta t)^2$$

$$\Delta t = \sqrt{\frac{-2\Delta d_y}{a_y}}$$

$$= \sqrt{\frac{-2(8.0)}{-9.8}}$$

$$= 1.3 \text{ s}$$

$$v_x = \frac{\Delta d_x}{\Delta t}$$

$$\Delta d_x = v_x \Delta t$$

$$= (12.0)(1.3)$$

$$= 16 \text{ m}$$

∴ Mike was 16 m away when he kicked the ball.

SUMMARY – ACCELERATION DUE TO GRAVITY

- Free fall is the motion of an object falling toward the surface of Earth with no other force acting on it than gravity.
- The average acceleration due to gravity at Earth's surface is $g = 9.8\ \mathrm{m/s^2}$ [down].
- The acceleration due to gravity depends on latitude, altitude, and local effects, such as the distribution of mineral deposits.
- The constant acceleration equations can be applied to analyze motion in the vertical plane.
- Terminal speed is the maximum speed reached by an object falling in air or other fluids. When a falling object reaches terminal speed, its downward acceleration becomes zero and its velocity becomes constant.

SUMMARY – PROJECTILE MOTION

- A projectile is an object moving through the air in a curved trajectory with no propulsion system.
- Projectile motion is motion with a constant horizontal velocity combined with a constant vertical acceleration.
- The horizontal and vertical motions of a projectile are independent of each other except they have a common time.
- Projectile motion problems can be solved by applying the constant velocity equation for the horizontal component of the motion and the constant acceleration equations for the vertical component of the motion.

PRACTICE

Readings

- Section 1.3 (pg 32)
- Section 1.4 (pg 41)

Questions

- pg 40 #6,7,8a,9a
- pg 50 #1-5,8,9